R22

Code No: 182AR

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year II Semester Examinations, February - 2025

ORDINARY DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, AE, MIE, CSIT, CE(SE), CSE(CS), CSE(AI&ML), CSE(DS), CSE(IOT), AI&DS, AI&ML, CSD)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Check the equation $(3x^2+2e^y)dx+(2xe^y+3y^2)dy=0$ for exactness. [1]

b) Find the integrating factor of $\frac{dy}{dx}-y\tan x=3e^{-\sin x}$. [1]

c) Solve the Cauchy-Euler differential equation $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-20y=0$. [1]

d) Find the particular integral of the differential equation $(D^2+9)\,y=\cos 4x$. [1]

e) Find the Laplace transform of $\sin^2(at)$. [1]

f) Find the inverse Laplace transform of $\frac{1}{s^2-4s+20}$. [1]

g) Find ∇f at $P(1,1,1)$, if $f=5x^2y-5y^2z+2.5z^2x$. [1]

h) Find b such that the force field $\overline{A}=(e^x z-bxy)\overline{i}+(1-bx^2)\overline{j}+(e^x+bz)\overline{k}$ is irrotational. [1]

i) Apply Green's theorem to prove that the area enclosed by a plane curve is $\frac{1}{2}\oint_C (x\,dy-y\,dx)$. [1]

j) State the Stoke's theorem. [1]

PART - B

(50 Marks)

2 a) Solve the differential equation $\sqrt{1-y^2}\,dx=(\sin^{-1}y-x)\,dy$.

b) According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is 290 K and the substance cools from 370 K to 330 K in 10 minutes, find when the temperature will be 295 K. [5+5]

OR

3.a) Solve the differential equations $\frac{dy}{dx} + y\tan x = y^3 \cos x$.

b) Radium disintegrates at a rate proportional to its mass. When mass is 10 mgm, the rate of disintegration is 0.051mgm per day. How long will it take for the mass to be reduced to 10 to 5 mgm? [6+4]

4.a) Find the general solution of the differential equation
$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x.$$

b) Solve the differential equation $\frac{d^2y}{dx^2} + 4y = 4\sec^2 2x$ by method of variation of parameters. [5+5]

OR

5.a) Find the general solution of the differential equation $\frac{d^4y}{dx^4} - y = e^x \cos x$.

b) A circuit consists of an inductance of 0.05 henrys, a resistance of 5 ohms and a condenser of capacitance 4×10^{-4} farad. If Q = I = 0 when t = 0, find Q(t) and I(t) when there is an alternating e.m.f. E(t) = 200 cos 100t. [5+5]

6.a) Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.

b) Find $L^{-1}\left\{\frac{5s^2 + 3s - 16}{(s-1)(s-2)(s+3)}\right\}$. [5+5]

OR

7.a) Using the first shifting theorem, find the Laplace transforms of the following functions. (i) $e^{-at}\cos bt$, (ii) $e^{at}\sin bt$.

b) Use Laplace transforms, Find the solution of the initial value problem
$y'' + 6y' + 13y = e^{-t}$, $y(0) = 0$, $y'(0) = 4$. [5+5]

8.a) Find the directional derivative of $f(x,y,z) = x^2y - y^2z - xyz$ at the point (1, -1, 0) in the direction of $\bar{i} - \bar{j} + 2\bar{k}$.

b) If $\bar{a}$ is a constant vector and $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$,

Prove that $\nabla \times \left[\frac{1}{r^3}(\bar{a} \times \bar{r})\right] = \frac{3}{r^5}(\bar{a} \cdot \bar{r})\bar{r} - \frac{\bar{a}}{r^3}$, $r = |\bar{r}|$. [4+6]

OR

9.a) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = |\bar{r}|$, show that $div\left(\frac{\bar{r}}{r^3}\right) = 0$.

b) Show that $Curl\ (Curl\ \bar{F}) = grad\ (div\ \bar{F}) - \nabla^2 \bar{F}$. [5+5]

10.a) Find the work done by the force $\overline{F} = x^2\,\bar{i} + yz\,\bar{j} + z\bar{k}$ in moving a particle from a point P to the point Q, where C is the line segment from P (1, 2, 2) to Q (3, 4, 2).

b) Using divergence theorem, evaluate the surface integral:
$\iint_S \left[x^3\, dydz + x^2 y\, dzdx + x^2 z\, dxdy\right]$, where S: closed surface consisting of the circular cylinder $x^2 + y^2 = a^2$, $(0 \le z \le b)$ and the circular disks z = 0 and z = b, $(x^2 + y^2 = a^2)$.

[5+5]

OR

11.a) Using Stokes' theorem, Evaluate $\oint_C [(x + y)dx + (2x - z)dy + (y + z)dz]$ where c is the boundary of the triangle with vertices $(2,0,0), (0,3,0)$ and $(0,0,6)$.

b) Use Green's theorem to evaluate the line integral $\oint_C (Mdx + Ndy)$ when $Mdx + Ndy$ equals to: $(3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c: boundary of the region defined by $x = 0, y = 0, x + y = 1$.

[5+5]

---ooOoo---